2018

(October)

MATHEMATICS

(Elective/Honours)

(GHS–11)

(Algebra—I and Calculus—I)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Find the domain of the function

$$f(x) = \frac{1}{\sqrt{|x| - x}}$$

3

(b) If

$$f(x) = \frac{1+e^x}{1-e^x}, \quad x \neq 0$$

show that $f(x)$ is an odd function. 3½

(c) A function f is defined as follows :

$$f(x) = 2x+3 \quad \text{for} \quad x > 2$$
$$= 3x+4 \quad \text{for} \quad x \le 2$$

Examine if $f(x)$ is continuous at $x = 2$. Draw the graph of $f(x)$. 3½+1=4½

(d) A and B are two sets as given below :

$$A = \{1, 2, 3\}, \; B = \{x, y\}$$

Obtain $A \times B$ and $B \times A$. 2+2=4

2. (a) Using the definition of limit at ∞, show that

$$\lim_{x \to \infty} \frac{x}{1+x} = 1$$ 4

(b) Let $f, g : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = e^x$ and $g(x) = \sin x$. Obtain $f \circ g$ and $g \circ f$. Is $f \circ g = g \circ f$? Discuss the continuities of $f \circ g$ and $g \circ f$. 3+½+1½=5

(c) A relation R is defined on $\mathbb{R}$, the set of real numbers, as follows :

aRb when $a \ne b$. Examine if the relation is (i) reflexive, (ii) symmetric and (iii) transitive. 3

(d) Prove that for any two sets A and B, $(A \cap B)' = A' \cup B'$, where A' = complement of A. 3

(Continued)

UNIT—II

3. (a) Give example of a mapping $f : A \to B$ such that f is—

(i) one-one but not onto;

(ii) onto but not one-one;

(iii) one-one and onto;

(iv) neither one-one nor onto. 4

(b) If A and B are two matrices such that $AB = A$ and $BA = B$, show that A' and B' are idempotent. ($A' \equiv$ transpose of A). 4

(c) Examine if the following system of equations is consistent and if so, find the solution : 7

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x + y - z = 1$$

4. (a) If A is a non-singular square matrix of order n, prove that $|adj\, A| = |A|^{n-1}$. 3

(b) Reduce the following matrix into normal form and hence obtain its rank : 8

$$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

(Turn Over)

(c) Prove that every square matrix is uniquely expressible as the sum of a Hermitian and a skew-Hermitian matrices. 4

UNIT—III

5. (a) Using definition, find the derivative of x^2+7x+9. 4

(b) Find $\frac{dy}{dx}$ (any *one*), when 3

(i) $y=\cos^{-1}\frac{1-x^2}{1+x^2}$;

(ii) $y=\frac{1-\sin x}{1+\sin x}$.

(c) If $y=x^{\tan x}+(\sin x)^{\cos x}$, find $\frac{dy}{dx}$. 5

(d) If the rate of change of y with respect to x is 5 and x is changing at 3 units per second, how fast is y changing? 3

6. (a) If $y=\sin^{-1}x$, prove that 5

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$$

(b) Evaluate any *one* of the following : 3

(i) $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{1-\cot x}$

(ii) $\lim_{x\to 1}\left[\frac{x}{x-1}-\frac{1}{\log x}\right]$

(c) When is a function said to be uniformly continuous in an interval? Show that the function $f(x)=x^2$ is uniformly continuous in $[-1, 1]$. 1+3=4

(d) Find the derivative of $x^{\sin^{-1}x}$ with respect to $\sin^{-1}x$. 3

UNIT—IV

7. (a) Evaluate any *one* of the following : 3½

(i) $\int\frac{1}{x(x+1)^2}dx$

(ii) $\int\frac{dx}{5+4\cos x}$

(b) Show that (any *one*) 3½

(i) $\int_e^{e^2}\frac{dx}{x\log x}=\log 2$;

(ii) $\int_0^{1/2}\sin^{-1}x\,dx=\frac{\pi}{12}-1+\frac{\sqrt{3}}{2}$.

(c) Show that

$$\int\tan^5 x\,dx=\frac{\tan^4 x}{4}-\frac{\tan^2 x}{2}+\log|\sec x|$$ 4

(d) Prove that

$$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\cdots+\frac{1}{8n}\right]=\frac{3}{8}$$ 4

8. (a) If

$$f(x) = \cos x \quad \text{for} \quad -\tfrac{\pi}{2} \le x \le 0$$
$$= \sin x \quad \text{for} \quad 0 < x \le \tfrac{\pi}{2}$$

show that

$$\int_{-\pi/2}^{\pi/2} f(x)\,dx = 2$$ 3

(b) Using the properties of definite integral, show that

$$\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin^2 x} = \frac{\pi^2}{2ab}$$ 4

(c) Examine the convergence of

$$\int_0^1 \frac{dx}{x^{2/3}}$$ 4

(d) Using the definition of definite integral, evaluate

$$\int_a^b e^x\,dx$$ 4

UNIT—V

9. (a) Show that the differential equation whose general solution is $y = ax + bx^2$ is

$$y = x\frac{dy}{dx} - \frac{1}{2}x^2\frac{d^2y}{dx^2}$$ 3

(b) Solve any *two* of the following : 2½×2=5

(i) $xy^2dy - y^3dx + y^2dy = dx$

(ii) $x^2dy + (xy + 2y^2)dx = 0$

(iii) $x\dfrac{dy}{dx} = y + \cos^{-1}\dfrac{1}{x}$

(c) Solve any *one* of the following : 3

(i) $x\log x\dfrac{dy}{dx} + y = 2\log x$

(ii) $x\dfrac{dy}{dx} = y + e^{1/x}$

(d) Solve any *one* of the following : 4

(i) $(1+y^2)dx - (\tan^{-1}y - x)dy = 0$

(ii) $\dfrac{dy}{dx} + \dfrac{y}{x}\log y = \dfrac{y}{x^2}(\log y)^2$

10. (a) Solve any *two* of the following equations : 3½×2=7

(i) $p^2 - p(x+y) + xy = 0$

(ii) $y = (1+p)x + p^2$

(iii) $y = yp^2 + 2px$

(*b*) Obtain the complete primitive and singular solution of

$$(y+1)p - xp^2 + 2 = 0$$ 4

(*c*) Find the orthogonal trajectories of the curve

$$x^2 + y^2 + 2gx + c = 0$$

where g is a parameter. 4

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